After several years of intense experimental and theoretical study of the origin of unusually high-temperature superconductivity\(^1\) in MgB\(_2\), nowadays there is a general belief that the superconductivity in MgB\(_2\) is phonon mediated with multiple gaps and an exceptional role played by \(E_{2g}\) phonon mode in strong coupling to electrons in the tubular-shaped \(\sigma\) bands at the Fermi level, \(E_F\). However, despite the tremendous progress achieved in understanding of superconductivity in MgB\(_2\), many of its fundamental properties related to phonons and electron-phonon (\(e-ph\)) coupling are still puzzling and remain under intense debate. Thus, such explanations remain puzzling and remain under intense debate. Thus, such explanations.

As for the electronic part of the \(e-ph\) picture of superconductivity in MgB\(_2\), it has received substantially less attention than the phonon part. In particular, in all evaluations of the \(e-ph\) interaction the adiabatic approximation\(^1\) of \(T_c\), the dynamical Coulomb interaction has been considered in its static form. Here we present a detailed \textit{ab initio} study of the low-energy dynamical electronic properties of MgB\(_2\). We demonstrate that the strongly anisotropic electronic structure of MgB\(_2\) characterized by boron quasi-two-dimensional \(\sigma\) and three-dimensional \(\pi\) bands\(^1\) leads to remarkable low-energy dielectric response in this compound: collective modes with a peculiar sine-like oscillating dispersion appear in the 0–0.5 eV energy range. This brings interesting physics to the energy region, which was thought to be entirely dominated by lattice vibrations.

Our approach is based on time-dependent density functional theory\(^1\), where the nonlocal dynamical density-functional response function, \(\chi\), determines an induced charge density, \(\rho\), in the electronic system caused by an external potential, \(\nu_{\text{ext}}\), according to

\[
\rho(r,t) = \int \chi(r,t;r',t') \nu_{\text{ext}}(r',t') dr' dt'.
\]

\(\chi\) is obtained from the integral equation

\[
\chi^\sigma_{\text{GG}}(t') = \chi^\omega(t) + \chi^\omega(t + E_{\text{nn}}^\sigma - E_{\text{nn}}, k, \mathbf{q}) \delta_{\text{nn}}(E_{\text{nn}} - E_{\text{nn}}, k, \mathbf{q} + \omega)
\]

\[
\times \langle \psi_{\text{nn}}(\mathbf{r} - \mathbf{q} + \mathbf{G}^\text{\(\sigma\)}, \mathbf{r}) | \psi_{\text{nn}}(\mathbf{r}) \rangle
\]

\[
\times \langle \psi_{\text{nn}}(\mathbf{r} - \mathbf{q} + \mathbf{G}^\text{\(\sigma\)}, \mathbf{r}) | \psi_{\text{nn}}(\mathbf{r}) \rangle
\]

and \(\text{Re}[\chi^\omega_{\text{GG}}(t', t)]\) is evaluated via the Kramers-Kronig relation using an energy cutoff of 50 eV. Here, the \(\mathbf{G}\)’s are the reciprocal lattice vectors, \(n, n'\) are band indices, the wave vectors \(\mathbf{k}\) and \(\mathbf{q}\) are in the first Brillouin zone (BZ), the factor 2 accounts for the spin, and \(\Omega\) is the normalization volume. In practice, we replace the \(\delta\) function in Eq. (2) by a Gaussian \(\frac{1}{\pi \gamma} \exp(-\gamma^2 x^2)\) with a very small broadening parameter \(\gamma = 10\) meV. The sum over \(\mathbf{k}\) was performed on a 108 \times 108 \times 360 grid. The use of such a fine \(\mathbf{k}\)-mesh sampling is crucial to achieve convergence of dielectric properties in MgB\(_2\) at low energies. The one-particle energies \(E_{\text{nn}}\) and wave functions \(\psi_{\text{nn}}\) are obtained as self-consistent solutions of Kohn-Sham equations using norm-conserving pseudopotentials\(^1\) and the exchange-correlation potential of Ref. 21. In order to elucidate the role of exchange-correlation effects in the density-response function, we performed calculations of \(\chi\) using two forms of the many-body kernel \(K_{\text{xc}}\), namely, the random-phase approximation (RPA) \((K_{\text{xc}}=0)\), and an adia-
FIG. 1. (Color) Calculated $-\text{Im}[\varepsilon^{-1}(q, \omega)]$ versus $\omega$ and $q_c$ in two energy regions. Calculations include local-field effects and the RPA kernel. In (a) the circles and squares mark the energy-loss peak positions measured in x-ray scattering experiments of Refs. 24 and 25, respectively. In (b) the dispersion of the upper sharp $\sigma\pi$ plasmon peak is described by $\omega_{\text{pl}}=V_{\text{br}}|\sin(\pi q_c)|$ (green dashed line) with $V_{\text{br}}=0.48$ eV. The lower broad feature with similar sine-like dispersion corresponds to the $\sigma\sigma$ plasmon. Yellow solid line presents acoustic plasmon dispersion according to Ref. 32.

The calculated loss function, $-\text{Im}[\varepsilon^{-1}(q, \omega)]$, directly probed in inelastic scattering experiments is presented in Fig. 1 as a function of the momentum transfer $q$ along the $c^*$ axis. In the upper energy range [Fig. 1(a)] it is dominated by a well-defined collective mode dispersing in the 2.5–4.5 eV energy range in excellent agreement with x-ray experiments.24,25 The existence of this mode for momenta in the first BZ was demonstrated in Refs. 26 and 27, whereas its cosine-like oscillating dispersion in higher BZ’s was recently discovered in a joint experimental-theoretical study.25 This mode produces a strong impact on electrodynamical and optical properties of MgB$_2$, however, it has no relevance to superconductivity as it affects neither the dynamically screened Coulomb interaction at low energies nor phonon dispersion.27

Figure 1(b) shows the calculated loss function in the low-energy domain that is our main finding. At all $q_c$ it is dominated by a sharp “$\sigma\pi$” peak with a sine-like dispersion. Figure 2 shows the dielectric and loss functions at $q_c=0.055$ a.u.$^{-1}$ and $q_c=0.322$ a.u.$^{-1}$. One can see how the presence of two types of carriers (in the $\sigma$ and $\pi$ bands) around $E_F$ [see Fig. 3(a)] with a large difference in the perpendicular component of the group velocity, $v^\perp$, [compare maximal $v^\perp$ in the $\pi$ bands [Fig. 3(b)] with that in the $\sigma$ bands [Figs. 3(c) and 3(d)]] produces in $\text{Im} \varepsilon$ a structure consisting of two main peaks [Fig. 2(a)]. Thus, while the faster $\pi$ carriers give rise to a broad structure from 0 to 0.68 eV in Fig. 2(a) with the main peak at the upper-energy side, the $\sigma$ carriers which are moving more slowly in the $c^*$ direction produce a sharp peak in the low-energy part of $\text{Im} \varepsilon$. The reason of this sharpness resides in the fact that within both $\sigma$

bands, the number of states with maximal $v^\perp$ is greatly enhanced as seen in Figs. 3(c) and 3(d). Therefore, the number of intraband transitions involving these fast states is large which strongly enhances the $\sigma$ peak in $\text{Im} \varepsilon$ at the higher energies. In turn, this causes a dramatic drop in $\text{Re} \varepsilon$ at nearly the same energy which, combined with the presence of a local minimum around $\omega=0.1$ eV in $\text{Im} \varepsilon$, produces a well-defined sharp peak in $-\text{Im}[\varepsilon^{-1}]$ corresponding to charge density fluctuations between the $\sigma$ and $\pi$ bands, a $\sigma\pi$ mode. Despite the presence of a nonvanishing value of $\text{Im} \varepsilon$ at the
where the $\sigma\pi$ peak appears, its intrinsic width is extremely small, being in the meV range (which corresponds to a lifetime of few hundreds of femtoseconds), and is almost entirely determined by an extrinsic broadening parameter $\gamma$.

Since at small $q_z$, the two main peaks in $\text{Im} \, \epsilon$ disperse linearly with momentum according to their maximal perpendicular Fermi velocity components, $\text{Im} \, \epsilon \propto v_{F,\text{max}}^{(\pi)} \cdot q_z$, the dispersion of the $\sigma\pi$ mode (linked to the upper side of the continuum for electron-hole excitations within the $\sigma$ bands) is also linear in $q_z$ and its group velocity $v_{\sigma\pi}$ is close to $v_{F,\text{max}}^{(\pi)}$. This corresponds to the acoustic plasmon proposed to exist in MgB$_2$ on the basis of a model tight-binding calculation. The concept of an acoustic plasmon goes back to Pines who showed that it can occur in a two-component electron plasma consisting of slow and fast carriers. The fast carriers (within the $\pi$ bands in MgB$_2$) can act to screen the repulsion between slow carriers (within the $\sigma$ bands in MgB$_2$) resulting in the appearance of a plasmon mode with a peculiar sound-like dispersion. After the recognition of the role in the acoustic plasmon as a possible mechanism for superconductivity in transition metals, it has been regularly evoked for explaining superconductivity in materials with unusually high $T_c$. Nevertheless, up to now the issue remains controversial since the very existence of such a collective mode in metals has been confirmed neither experimentally nor by $ab$ initio calculations. Only recently an acoustic-like plasmon was observed in the electron energy-loss measurements at a metal surface in excellent agreement with the $ab$ initio prediction, thus greatly increasing our confidence in the present results.

With increasing $q_z$, the $\sigma$ peak in $\text{Im} \, \epsilon$, and consequently the $\sigma\pi$ peak in $-\text{Im} \, [\epsilon^{-1}]$, starts to split into two peaks [Figs. 1(b) and 2(b)] since for holes, $v_{\sigma\pi}^{(\pi)}$ exceeds $v_{\tau\pi}^{(\tau)}$ by more than 20% [Figs. 3(c) and 3(d)]. In this case the higher (lower)-energy plasmon peak in $-\text{Im} \, [\epsilon^{-1}]$ corresponds to the charge fluctuations between $\sigma_1$ and $\pi$ ($\sigma_2$ and $\sigma_2$) bands. Whereas the $\sigma\pi$ plasmon continues to be long-lived, the lower-energy $\sigma\pi$ plasmon has a significantly shorter lifetime due to more efficient scattering within the $\sigma_1$ band. We expect the two separate modes to exist at any $q_z$, however, when the calculated broadening $\gamma$ exceeds the energy difference between these modes only a single $\sigma\pi$ peak arises in $-\text{Im} \, [\epsilon^{-1}]$ [Fig. 2(a)].

In Fig. 1(b) one can see how with increasing $q_z$, both of these plasmon modes reach maximum energy at the A point and, as momentum increases further, their dispersions change from positive to negative and approach $\omega=0$ at the $\Gamma$ point in the second BZ, in striking contrast to results of Ref. 32. The origin of this periodic behavior resides in the fact that strong local-field effects in MgB$_2$ feed the strength from the small $q_z$ modes into the charge density fluctuations at large $(q_x+G_x)$ in a similar fashion as occurs in the case of the higher energy mode. These sine-like dispersing modes continue to have strength at momenta in subsequent BZ’s and this is a direct consequence of a layered MgB$_2$ structure. Additionally, as seen in Fig. 2, the local-field effects lead to a blue shift of the $\sigma\pi$ and $\pi\tau$ energy and make the $\sigma\pi$ peak more narrow as it occurs for $\omega$ corresponding to smaller $\text{Im} \, \epsilon$. We also analyzed the role of exchange-correlation effects beyond the RPA and found that the RPA picture is essentially sufficient for the description of low-energy collective excitations in MgB$_2$. The inclusion of the TDLDK kernel in the calculation of $\chi(q_z,\omega)$ leads to only a small (few percent) downward shift of the plasmon mode dispersions with a slight reduction of their lifetimes, i.e., in part compensating the local-field effects.

The $\sigma\pi$ plasmon can have deep impact on both low-energy-electron and phonon dynamics in MgB$_2$. Our calculation reveals dramatic modification of the dynamical Coulomb interaction in the energy range vital for superconductivity. In Fig. 2 one can see that in the neighborhood of the $\sigma\pi$ plasmon energy the calculated $\text{Re} \, [\epsilon(q_z,\omega)]$ differs dramatically from static $\epsilon(q_z,\omega=0)$ as well as from the free-electron gas result. In particular, in a certain momentum-energy phase space region, $\text{Re} \, \epsilon$ is even negative leading to overall reduction of Coulomb repulsion between carriers. This could explain in part the observed reduced isotope effect in MgB$_2$. It would be of great interest to quantify this effect within the $ab$ initio approach. Note that the $\sigma\pi$ plasmon does not influence interband $\sigma-\pi$ and intraband $\sigma$ scattering due to the phase-space restrictions, although could have some effect in the intraband $\pi$ one.

The $\sigma\pi$ plasmon can dramatically affect the dispersion of the optical phonon modes in the “pathological” region of small $q_z$, offering an unexpected explanation of widely discussed discrepancies between the Raman and x-ray measurements of the phonon structure in MgB$_2$. The $\sim 77$ meV peak observed in Raman-scattering experiments and commonly attributed to the boron $E_{2g}$ phonon mode is always strongly renormalized in comparison with a bare phonon dispersion, whereas the x-ray experiments performed for large $q_z$ do not see any appreciable renormalization. In Fig. 4, we show the
calculated dispersion of the bare $\sigma\pi$ mode along with the bare $E_{2g}$ phonon mode compared to the experimental data. One can see how the two bare curves cross each other close to the 1 point. Due to its localization in the boron plane, the $\sigma\pi$ plasmon strongly interacts with the boron $E_{2g}$ phonon mode resulting in strong hybridization of these boron modes. The result shown by green and blue dashed lines in Fig. 4 demonstrates that the mode seen in the Raman experiments is indeed a strongly hybridized $\sigma\pi-E_{2g}$ mode. Note, that the finite electron lifetime effect\cite{8,13} can also contribute to the upward shift of this mode. Additionally, a steep $\sigma\pi$ plasmon dispersion at small momenta might explain the existence of the strong unstructured background (commonly mentioned as being of unknown electronic origin) observed in the Raman experiments. This is corroborated by the fact that, e.g., in AlB$_2$, where we do not expect such a plasmon, this background does not appear.\cite{5,8}

In conclusion, our detailed $ab\ initio$ calculation demonstrates the existence in MgB$_2$ in the 0–0.5 eV energy range of hitherto unknown long-lived collective mode that corresponds to coherent charge fluctuations between the boron $\sigma$ and $\pi$ bands ($\sigma\pi$ mode) with striking periodic sine-like dispersion. This mode shows an acoustic-like behavior at small momenta where it can strongly interact with optical phonons. Additionally we find at slightly lower energy a more strongly damped mode that corresponds to charge fluctuations between two different $\sigma$ bands ($\sigma\sigma$ mode). Both these modes have profound impact on the low-energy dynamical Coulomb interaction which should be explicitly taken into account in $ab\ initio$ theories of superconductivity to have the predictive power.

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23. $\epsilon_{\text{R}}^{\text{R}}=\epsilon_{\text{R}}^{\text{R}}(q, \omega)=1+4\pi\epsilon(1+G^{\text{GG}}(q, \omega))$ evaluated with the use of 25 G vectors.
31. In the present numerical calculations this has been demonstrated down to the energy $\omega \sim 25$ meV.
Thus, the usage of $\gamma=100$ meV leads to the occurrence of a broad single $\sigma\pi$ peak at all $q_c$ only. A. Balassis, E. V. Chulkov, P. M. Echenique, and V. M. Silkin, Phys. Rev. B 78, 224502 (2008).